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ABSTRACT

A study designed to investigate the association between Piagetian cognitive and long-term mnemonic (recall and recognition) levels and problem solving in mathematics was conducted with 152 eighth-grade students. Cognitive level was determined by two evaluations (seven months apart) of a subject's performance on a clinically administered balance beam task. Mnemonic level was assessed clinically by measuring a subject's memory of three geometric figures taken two at a time. Problem solving performance was measured by the score achieved on the Advanced Metropolitan Problem Solving Test, administered at the same time as the second balance beam task. Loglinear and path analysis models were used to describe the association. Data support the link between Piagetian cognitive and long-term mnemonic levels and a measure of school mathematics achievement, problem solving. Results show that cognitive level and its differentiated aspects as determined by long-term mnemonic measures are important in determining problem solving success. Cognitive level and long-term mnemonic level are both highly correlated with problem solving separately. Although they are highly correlated with each other, they provide additional information about problem solving level when jointly viewed. (Author/RL)

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The Association Between Piagetian Cognitive and Long-Term Mnemonic Levels and Problem Solving in Mathematics

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Abstract

A study designed to investigate the association between Piagetian cognitive and long-term mnemonic (recall and recognition) levels and problem solving in mathematics was conducted with 152 eighth-grade students. Loglinear and path analysis models were used to describe the association. The data support the link between Piagetian cognitive and long-term mnemonic levels and a measure of school mathematics achievement, problem solving. The results show that cognitive level and its differentiated aspects as determined by long-term mnemonic measures are important in determining problem solving success. Cognitive level and long-term mnemonic level are both highly correlated with problem solving separately. Although they are highly correlated with each other, they provide additional information about problem solving level when jointly viewed.

The Association Between Piagetian Cognitive and Long-Term Mnemonic Levels and Problem Solving in Mathematics

According to Piagetian memory theory, cognition and memory may be viewed as an interrelated system (Piaget and Inhelder, 1973). In a study designed to investigate the relationship between cognitive and mnemonic levels, Maher and O'Brien (1980) supported the relationship and particularized it so as to indicate that long-term recall was contingent upon cognitive level at the time of memory retrieval. More specifically, they showed that the improvement of memory over time was accounted for by advance in cognitive level or alternatively by a lateral development in a stable cognitive level.

Surprisingly, a search of the literature fails to uncover studies directed to the determination of the association, if any, between cognitive/mnemonic levels and problem solving in mathematics although the question of problem solving is receiving a great deal of attention from mathematics educators.

Despite the interest in and the efforts directed to its resolution, there is yet to be established a consensus on the nature of problem solving skill. However, there is emerging general recognition that it is rather more complex than has been envisioned.

The establishment of a theoretical framework explaining or describing problem solving ability in mathematics would generate hypotheses capable of detailed empirical study of it. Nevertheless, there presently exist instruments, however crude, that are commonly regarded as disclosing measures of the existence of problem solving ability in mathematics. It is one such instrument that is used here.

Thus this study sets itself the relatively limited objective of determining whether by existing measures of it an association can be iden-

tified between cognitive/ mnemonic levels and problem solving ability in the expectation that such a determination must be made before more refined and precise studies can be undertaken.

Soviet psychologist V. A. Krutetskii has devoted many years to the study of mathematical abilities in schoolchildren. A hypothesis underlying Krutetskii's research is that the ability to learn mathematics, that is problem solving ability, has two components: the ability to generalize mathematical material and the ability to remember the generalizations (Krutetskii, 1976).

On the ground that the ability to generalize mathematical relations and operations implies the capacity for formal thought and remembering the generalizations implies successful long-term recall memory, the association of these two components with problem solving is examined in this research.

Hence, the objective of this study is to investigate the relationship of these two variables, cognitive level and mnemonic level, to problem solving in mathematics both individually and jointly. These relationships are illustrated in the diagram presented in Figure 1.

Insert Figure 1 about here

Method

Subjects

The population of the study consisted of all 152 eighth-grade students in a suburban central New Jersey middle school district.

Procedures

Cognitive level was determined by evaluation of a subject's per-

formance on a clinically administered balance beam task and classified as belonging to one of four levels as described by Inhelder and Piaget (1958). The task involves balancing a beam with two weights by taking into account the distance of each from the fulcrum. Thus, an understanding of the proportionality between weight and distance was necessary for success. According to Piaget, the accomplishment of this task requires the achievement of formal thought operations. At seven months, the balance beam task was again administered in order to monitor intervening cognitive growth, if any, and explain its effect on memory.

Using the procedures outlined by Piaget and Inhelder (1973), corresponding mnemonic level was assessed clinically by measuring a subject's memory of three geometric figures taken two at a time. Piaget and Inhelder hold that the understanding and memory of the permutation calls for a group of higher order structures based on formal thought (1973). Recall memory drawings were obtained immediately and again at six-months. In addition, each subject was given a recognition task at the final (six-month) interview. For recall, subjects were classified as belonging to one of the four mnemonic stages or their substages paralleling the four stages of Piaget and Inhelder (1973) and for recognition, to one of four.

Protocols were developed to parallel cognitive and mnemonic levels and were field tested over a range of three grades - seventh, eighth, and ninth and revised.

Clinicians were carefully trained by experienced interviewers. When their interview performance was the same as that of the trainers, they were assigned to conduct interviews for the study.

All tapes and records for the balance beam task were reviewed by a committee of researchers. In instances of disagreement, the independent assessments were then compared by a committee consisting of two researchers

and a third clinician. Inter-rater reliability was .96 on the balance beam task.

The memory drawings were rated independently by two researchers according to the mnemonic types described by Piaget and Inhelder (1973). All ratings were compared by the independent raters in committee with a third clinician. Inter-rater reliability for immediate recall memory was .98 and for six-month recall memory, .95.

The recognition task involved a choice by the subject of one drawing from among four, only one of which was correct. No errors were found in the recording of this task.

Problem solving performance was measured by the score achieved on the Advanced Metropolitan Problem Solving Test, Form F, administered in April at approximately the same time as the administration of the second balance beam task. The tests were machine scored by the test publishers and problem solving scores were categorized as high (at or above grade level) or low (below grade level).

The content of the test included (a) 4 one step addition, subtraction, multiplication and division problems, (b) 9 fraction and decimal problems, (c) 1 general solution problem, (d) 4 multiple step problems, (e) 3 geometry problems, (f) 3 ratio problems, (g) 2 percent problems, (h) 1 problem using negative integers, (i) 3 problems requiring the reading of charts, and (j) 5 number sentence problems.

Results

The data are described in the three-way classification array given in Table 1 consisting of each level and combination of levels of the variables cognitive level (A), long-term mnemonic level (B), and problem solving level (C)

Insert Table 1 about here

The association between cognitive, long-term mnemonic, and problem solving levels was described by statistical models of the odds of high problem solving conditioned on a subject's cognitive and long-term mnemonic levels and presented graphically with a path analysis model. All estimates of conditional odds and odds ratios are based on a loglinear model analysis of the cross-classification presented in Table 1 which gives the number of experimental subjects within each of the cognitive, long-term mnemonic, and problem solving categories. Maximum likelihood estimation procedures as described by Haberman (1978) were used. Use of the screening and stepping procedures described by Brown (1979) resulted in the selection of the simplest model given by Equation 1 that is consistent with the data. The model explains 92% of the variation not accounted for by the model assuming that problem solving level is independent of cognitive level and long-term memory level.

$$\log m_{ijk} = \lambda + \lambda^{A}_{i} + \lambda^{B}_{j} + \lambda^{C}_{k} + \lambda^{AB}_{ij} + \lambda^{AC}_{ik} + \lambda^{BC}_{jk} \qquad (1)$$

The odds in favor of a given event, problem solving for example, are dependent only on those variables in the model interacting with it. Odds for a given category (problem solving, for example) are the ratio of the number of high to low subjects within a group (here, formal and nonformal or recall and no memory). The expected cell counts in Table 1 are used to derive the 95% confidence interval estimates of the odds of high problem solving as a function of the subject's long-term memory and

cognitive level.

Relative odds with respect to a given event, problem solving for example, express the ratio of the odds in one group, recall for example, to the odds in another, no memory for example.

Using the expected cell counts in Table 1, the 95% confidenct interval estimates for the relative odds of high problem solving level as a function of (a) long term mnemonic levels and (b) cognitive levels were derived. These are given in Table 2.

Insert Table 2 about here

Table 2 shows that regardless of cognitive level, a subject with a high recall memory assessment has 1.7 times the odds of high problem solving level than a subject who has recognized at six-months. A subject with a high recall assessment has 4.5 times the odds of a high problem solving level as a subject without six-month memory. A subject who recognized at six-months has 2.6 times the odds of a high problem solving level as a subject without six-month memory. Additionally, Table 2 shows that regardless of long-term memory level, a subject with a high cognitive level assessment has 3.2 times the odds of a high problem solving level as a subject with a low cognitive level assessment.

The associations among cognitive level (A), long-term memory level (B), and problem solving level (C) described in Tables 1 and 2 are represented graphically in the Path Analysis of Figure 2.

Insert Figure 2 about here

All pairs of assessments that are conditionally correlated are con-

nected by paths. The magnitude of the conditional association between any two variables, say A and B, given a fixed assessment for a third variable C, is measured as the ratio of the conditional odds of a high variable A outcome given a high variable B outcome to the conditional odds of a high variable A outcome given a low variable B outcome. The conditional odds ratios or relative odds between A and B are shown in Figure 2 as the path weights between A and B. In general, the relative odds between two variables are symmetric and this is indicated by the double headed paths in Figure 2.

The AC path is to be interpreted as meaning that given a fixed outcome of the other conditioning variable (here, long-term memory level) a subject who was formal has 3.2 times the odds of a high problem solving assessment as a subject who was non-formal. A subject who recalled at six-months has 12.2 times the odds of a formal cognitive level assessment as a subject who did not remember. A subject who recognized at six-months has 1.02 times the odds of a formal cognitive level assessment as a subject who did not remember. A subject who recalled at six-months has 11.9 times the odds of a formal cognitive level assessment as a subject who recognized at six-months.

Similarly, the BC paths indicate that given a fixed outcome of the other conditioning variable (cognitive level) a subject who recalled at six-months has 1.7 times the odds of a high problem solving level as a subject who recognized. A subject who recalled at six-months has 4.5 times the odds of a high problem solving level as a subject who did not remember at six-months. A subject who recognized only has 2.6 times the odds of a high problem solving level as a subject who didn't remember.

A further advantage of the Path Analysis Model is that the product of the appropriate path weights or their reciprocals can be used to

compare the conditional odds ratios for different levels of two or more conditional variables. For high assessment levels, the product of the path weights are used; for low levels, the reciprocal of the the path weights are used.

The AC and appropriate BC paths jointly indicate that a formal subject who recalled at six-months has 14.4, (3.2×4.5), times the odds of a high problem solving level as a subject with corresponding low assessments. Examination of the BC path weights with the AC path weights in Figure 2 reveals the following: a subject with a formal cognitive level and successful six-month recall has 5.4,(3.2×1.7), times the odds of a high problem solving level as a subject with a non-formal cognitive level assessment and a six-month recognition memory assessment; a subject with a formal cognitive level assessment and who recognized at six-months has 8.3,(2.6×3.2), times the odds of a high problem solving level as a subject who did not remember.

A subject who was non-formal and did not remember at six-months has .07,$(\frac{1}{3.2} \times \frac{1}{4} \frac{1}{5})$, times the odds of a high problem solving level as a subject rated formal and who recalled at six-months. For a subject with a non-formal cognitive level assessment and recognition memory assessment, the odds of a high problem solving level is .18,$(\frac{1}{3.2} \times \frac{1}{1.7})$, times the odds as a subject with corresponding high assessments. Continuing to focus on the AC and BC paths, it can be observed that subjects with non-formal cognitive and no long-term memory assessments have .12, $(\frac{1}{3.2} \times \frac{1}{2.6})$, times the odds of a high problem solving as subjects rated as formal on cognitive level and who recognized at six-months. A subject with a non-formal cognitive level assessment and a who recognized has .8,$(\frac{1}{3.2} \times 2.6)$, times the odds of a high problem solving level as a subject with a formal cognitive level assessment and no six-month memory.

The AB path reveals the magnitude of the association between cognitive level and long-term mnemonic assessments found by Maher and O'Brien (1980).

Discussion

At least two necessary conditions for problem solving success are indicated: formal operations status and capacity for recall memory. Each contributes individually to problem solving achievement; both coexisting substantially increase the odds in favor of problem solving success.

The data disclose problem solving success among subjects either not possessing these two qualifications or possessing them in atypical combinations. Of the 152 subjects, 7 had low cognitive and mnemonic levels and high problem solving levels; 5 had high cognitive and mnemonic levels but low problem solving levels. There are several possible explanations. One is that the instrument employed for measuring success in problem solving includes items that require no more than elementary familiarity with certain fundamental operations that are ordinarily committed to memory and require no procedures that may be regarded as falling into a problem solving category. The instrument distinguishes only correct from incorrect answers. Thus, correct answers arrived at accidentally or by mechanical means rather than that recognition of the structure of the problem might suggest are indistinguishable. Also, there are factors entering into some successful problem solving that are as yet unidentified.

The necessary condition for developing problem solving ability in students is a clear theoretical description or definition of the problem solving process in order that the abilities, knowledge and procedures it requires may be identified as also their respective contributions to its

successful performance.

The hypotheses generated by the theory may then be submitted to empirical studies whose outcomes might suggest curriculums, teaching methods and mathematics program organization as might develop problem solving abilities in students.

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TABLE 1

Observed and Expected Frequencies with Ninety-Five Percent Confidence Intervals of the Conditional Odds for High Problem Solving

Memory	Cognitive Level	Cell Counts for Problem Solving: Low	High	Odds for High Problem Solving
Recall	Formal	4.7(5)[a]	21.3(21)	4.5 $^{+7.6}_{-2.8}$
	Non Formal	1.3(1)	1.7(2)	1.4 $^{+12.4}_{-1.2}$
Recognition	Formal	7.2(9)	18.7(17)	2.6 $^{+3.5}_{-1.5}$
	Non Formal	22.8(21)	18.3(20)	0.8 $^{+.7}_{-.4}$
No Memory	Formal	9(7)	9(11)	1.0 $^{+1.5}_{-.6}$
	Non Formal	29(31)	9(7)	0.3 $^{+.3}_{-.2}$

[a]Numbers in parentheses indicate the observed values.

TABLE 2

Ninety-Five Percent Confidence Interval Estimates of Relative Odds of High Problem Solving

Groups Being Compared	Relative Odds for High Problem Solving
Recall vs. Recognition	$1.7^{+3.7}_{-1.2}$
Recall vs. No Memory	$4.5^{+10.1}_{-3.1}$
Recognition vs. No Memory	$2.6^{+3.0}_{-1.4}$
Formal vs. Nonformal Cognitive Level	$3.2^{+3.6}_{-1.7}$

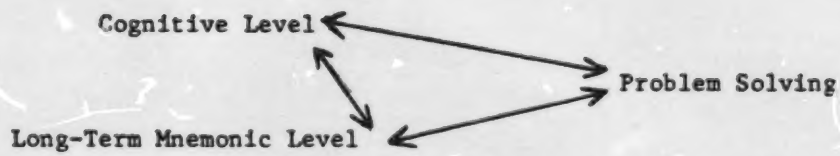

Figure 1. Relationship of Cognitive Level to Problem Solving and Mnemonic Level to Problem Solving.

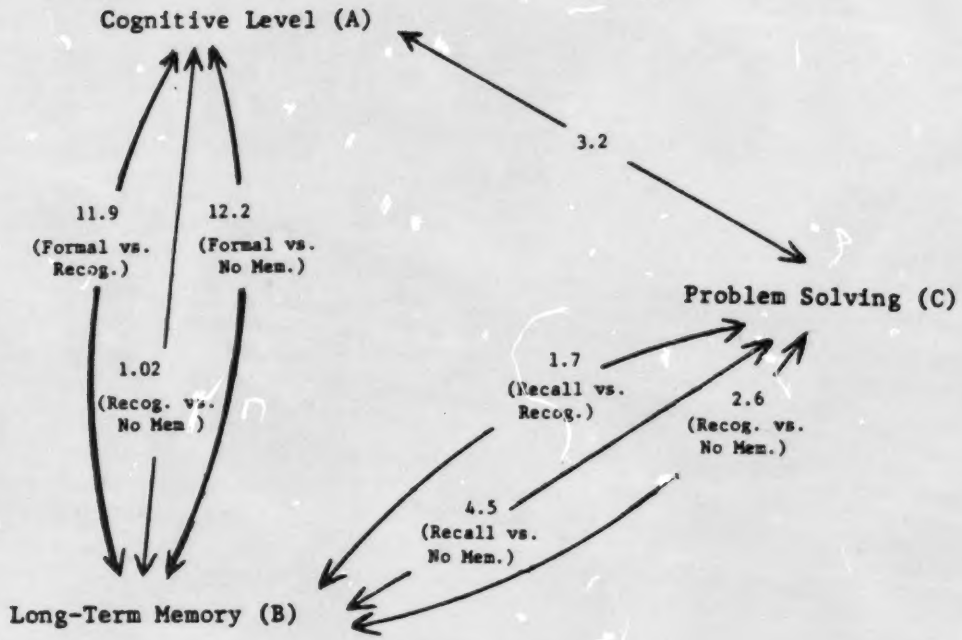

Figure 2. Path Analysis Illustrating the Association Between Cognitive and Long-Term Memory Levels and Problem Solving Level.